Forecasting Fund Flows and Liquidity Imbalances

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Abstract

In turmoil periods, market liquidity can experience sudden dry ups connected with significant price movements. This unexpected change in liquidity patterns, often driven by irrational investors' behavior, is normally defined as Liquidity Black Hole (LBH). So far relevant research in this area explored macro-market level interactions rather than micro-agent decision making processes.

In this study we show - both theoretically and empirically - that the LBH effect at market micro-level is originated by agents' decisions made at mutual fund level. We present a model of investors' behavior based on heterogenous expectations of market risk and return. The causes of a LBH are analyzed and the model is also applied to a specific mutual fund setting where leverage is allowed, but shortening the asset is forbidden (i.e. real estate mutual funds). Price creation is modeled both endogenously and exogenously. We show that the relationship between fund flows and expected liquidity risk follows an exponential function. Finally, we demonstrate that areas of absolute LBH exist and cannot be hedged. In those areas neither the available "cash-like cushion" nor the managerial skills of the market maker can avoid the "economic failure" of a fund.

1 Introduction

Liquidity is becoming a key phenomenon to explain the development of asset pricing and market movements. Over the last decade several studies have addressed the relationship between asset pricing and market movements. Some empirical studies, for example Goetzman and Ivkovic (2001), demonstrate the performance-driving effect of flows on mutual fund pricing, focusing on the difficulties in price determination due to international investment and differences in liquidity levels of underlying assets. Mutual funds, in particular, represent an interesting case study for such issues because they are characterized by inflows and outflows that are not necessarily and instantaneously matched, showing temporary imbalances in liquidity. Among others, two main determinants of liquidity have been identified in the literature: price signalling when investors reveal future price movements in their investment/disinvestment activities; return chasing behavior when market players read into past fund performances and take subsequent investment decisions on the basis of momentum or contrarian strategies.

In explaining the reasons behind liquidity movement and pricing, the behavioral aspects are by far the most interesting ones. Investors fear illiquidity for various reasons such as the possibility of facing personal liquidity shocks and consequently being forced to liquidate in a bearish market environment; the tendency to move the asset allocation of a portfolio from illiquid to more liquid investments when a liquidity crisis is approaching (i.e. flight to quality); the possible redemption of stocks at a price below expected values; and denial of redemptions due to wing up solutions or the presence of insufficient resources. This latter phenomenon is known as Liquidity Black Hole (i.e. LBH) and was first illustrated by Morris and Shin (2004), the same phenomenon has also been called liquidity run, liquidity spiral and flight to quality. Substantially, a LBH represent a run to liquidity due to a shock in the market. This shock is originated by the fear of an event or by a violation of specific conditions. Moreover, if the performance and price of an asset are determined by the behavior of other agents, investors can be influenced - contagion - by the general behavior, even if their individual conditions are not violated. This run to liquidity causes a sudden and deep change in the market driven by a strongly one-sided flow of money (i.e. outflow). The immediate effect is a sudden dry-up of liquidity, with consequences on the pricing of such assets. Metaphorically, it is as if the liquidity is completely sucked up by an invisible hole, exactly as it happens in astrophysics for black holes and matter.

In this paper we want to study the general problem of Liquidity Imbalances. In the literature, the imbalances constitute a critical phenomenon to be observed only if negative, and can lead to extreme events (i.e. LBH) only if significantly negative.

We apply the LI concept to mutual funds and study the process in which a LI is formed, identifying the main driving factors, its possible causes, determining the extent to which each single factor contributes to the formation of an imbalance. Moreover we account for LI of significant magnitude that can ultimately lead to extreme phenomenon.

Starting from the investment pattern of mutual funds, the origination of inflows and outflows is studied. The optimal behavior of an investor is examined as a function of both exogenous and endogenous market events. Through this procedure we are accounting for evidences of sources for the origination of a LI. This study will also then be able to raise some concerns about policy issues and implications and needs of self-regulation in mutual fund markets. Depending on the magnitude of the outflows, a LI can originate various phenomenon such as liquidity black hole (LBH) or fire sales. For this reason we will introduce and explain these effects, but we will account for this possibility only as a specific case of the more general Liquidity Imbalance case[1].

The price is first obtained starting from the normal interaction between buyers and sellers (i.e. market clearing conditions). Subsequently, the price is exogenously imposed by the fund manager. The main difference of the two approaches are considered and analyzed. The fund manager and market maker can react to the creation of a LI, and therefore modify the likelihood of a liquidity run. More specifically, the manager can interact with the idiosyncratic risk and can use some instruments or policies to minimize the impact. However for higher values of idiosyncratic risk there are no possibilities of hedging against losses and therefore extreme events of liquidity imbalances such as a liquidity black hole, can be originated. Anyway, as Hawkings' radiation theory suggests, the black hole reduces its energy over time and it disappears in the long-run, bringing values back to pre-hole conditions. The time for a complete recovery depends on the magnitude of the hole, and on the wealth of the market. Hameed, Kang, and Viswanathan (2010) demonstrate that a LBH lasts 1-2 weeks on average in the US equity market. The period is identified by external intervention (e.g. government) deemed to pre-empt a deterioration of the situation through injection of liquidity in the system. We argue that this timing depends on the underlying assets' liquidity, with less liquid markets (e.g. real estate) showing longer

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[1]Both LBH and fire sales are liquidity imbalances. The difference with general LI simply states on the consequences brought by these on the performance and the survivorship of a fund.

periods than more liquid ones (e.g. equity).

For this reason, after presenting a theoretical model, we use the real estate mutual fund (i.e. REMF) industry to run an empirical analysis on an existing fund in order to test the ability of our theoretical model to predict market phenomena. During the most recent period, in fact, we have noticed a steep rise of REMFs market performance, followed by a sudden decline in prices due to the presence of a very high liquidity in the early-mid 2000s and a sudden liquidity dry-up from the second half of 2007.

Several motivations are behind this choice. Firstly, the liquidity issue is more relevant than for other funds due to the very illiquid nature of underlying assets. When there is a sudden increase of unit redemptions (i.e. outflows), fund managers cannot easily sell assets in a short space of time (normally it takes 6 to 9 months to complete a transaction in real estate markets) and hence they try to maximize inflows to match potential outflows. On one hand, this feature could cause acceleration in the LI phenomenon - see Huang and Wang (2010) - due to the lack of actions able to generate immediate cash. However, on the other hand, this type of assets could attract more long-term conservative investors, who may be more able to absorb liquidity shocks in the short run. Secondly, a recent paper by Marcato and Tira (2010) using a panel VAR model demonstrates that inflows and out-flows reflect different behavioral attitudes of investors (respectively return chasing and pricing signal). Hence there is asymmetric information content between flows in and out of the fund. Even if in our model we do not model this asymmetry and treat buyers and sellers similarly, a natural extension of our model could incorporate this new assumption and theoretically find solutions explaining different behavioral attitudes. A third reason for using REMFs lies on the fact that they represent one of the very few industries where the fund manager and market maker coincide. The manager of an open ended REMF determines the bid-ask spread of the unit and is obliged to provide and redeem units to investors asking for it. As already mentioned above, since the secondary market is limited and it is mainly managed by in- and out-flows matching, market makers take decisions in relation to the fund performance and they are not only interested in widening the bid-ask spread. This co-participation provides a considerable change in the utility function of the market maker. Vayanos (2004), for example, demonstrates that the market maker is not simply risk averse and subjected to a problem of maximization of the bid-ask spread. In this context, the utility problem for the market maker becomes the maximization of the

bid-ask spread under the optimization of the manager's utility function (i.e. maximization of net flows). Furthermore the market maker is averse to the LI because it reduces the performance of the fund, while in the existing liquidity literature, the market maker is only considered in a LI problem for its inventory skills. In the classical approach, he is not damaged by imbalances because he simply adjusts the inventory and changes prices in response to investors' choices having a long-term investment horizon.[2] Fourth, the execution order in REMFs follows a FIFO (first in first out) rule. Therefore the possibility of redemption is dependent upon the decision of all other investors in the fund. This feature can create panic and ultimately originate a LBH as Bernardo and Welch (2004) demonstrate[3]. If an investor is willing to redeem but is aware that the likelihood of redemption depends on the decision of other investors, her fear of future liquidity shocks may lead her to redeem before the possibility of a liquidity run, causing the liquidity run itself. It is also true however that even equity mutual funds sometimes are not perfectly sequential in their execution orders. This would be an explanation of why a LBH could also be originated in stock markets. Following this assumption we decide to develop our model for the overall mutual fund industry and to use the real estate case only for empirical purposes.

Finally, since we consider the existence of a cost of participation in the market, a phenomenon allude to by Huang and Wang (2009, 2010), the illiquidity is originated by a non-full agents' participation - these costs (e.g. brokerage, fund raising, entry, management and redemption fees) tend to be higher for REMFs than for other industries.

Our paper brings a new contribution to the existing literature as it represents the first attempt to model a LI and his consequences in a mutual fund context. Previous literature analyzed the liquidity imbalances in a general financial market contest focusing on agents inside that market. The driving factors are normally derived from the general market condition (often simplified in a single-asset market) and this approach does not consider that, although a LBH may exist, there may also be some very liquid instruments. Consequently, we argue that a LI first originates at a fund level and then can spread to other funds or markets through a contagion effect. At the same time, we may find evidence of a global negative economic situation increasing the likelihood of a LI in many funds, without how-

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[2]However, Huang and J.Wang (2010) demonstrate that, under precise assumptions, the market making sector can hedge against the problem of liquidity shocks.

[3]In a further piece of research (i.e. the last part of my Ph.D.), we demonstrate that the optimal allocation of a single investor depends on the decision of other investors in the market

ever providing the concrete evidence of a contagion effect. Klaus and Rzepkowski (2009) expore the spillover effect on hedge funds and find that a liquidity run can occur for both internal redemption issues and through contagion effect from other funds. They show that bad market conditions are more likely to drive a consistent redemption flow in the fund rather than a contagion effect in the market.

The structure of the paper is the following: section 2 presents the relevant literature; section 3,4,5 respectively present the theoretical model, the relative market equilibrium and the optimal investment decisions. Section 6 discusses the application of the model to a real case scenario.

2   Literature review

Liquidity risk is becoming more relevant in asset pricing and particularly in real estate markets as Brounen, Eichholtz, and Ling (2009) demonstrate. They define liquidity as the speed of sale and price impact. They describe three measures of liquidity as trading (connected with selling speed), turnover and illiquidity. They show that the market capitalization is an important variable to define liquidity because of the higher volume connected with specific assets. Finally they discover a relationship between liquidity and asset pricing: the smaller the bid-ask spread is and the faster the movement of the security is (i.e. higher liquidity).

On one hand several papers focus on the effect of liquidity on asset pricing within stock and real estate markets, as well as for specific instrument (e.g. Brounen, Eichholtz, and Ling (2009), Marcato and Ward (2007), Subrahmanyam (2007), Bollen, Smith, and Whaley (2004), Hameed, Kang, and Viswanathan (2010), Aragon (2007), Sadka (2010)). Particularly Allen and Carletti (2008) demonstrate that during downturns, such as the current credit crunch, prices exchanged in the market are more representative of asset liquidity than future payoffs. Furthermore they show that prices are a function of liquidity, which can modify the asset performance. Finally, some articles specific to the real estate sector demonstrate the significance of liquidity in the pricing of assets such as REMFs - e.g. Marcato and Tira (2010), Gullet and Redman (2005), Tomperi (2009), O'Neal and Page (2000).

On the other hand there is an abundant literature on the effect of liquidity on investors' decisions. The behavioral finance/economics literature concentrates on both the origination of such phenomenon and its solutions. The origination of the decision process differs between studies: some present the fear for the worst scenario as the dominant effect. This fear causes liquidity imbalances or a LBH, which is represented by a significant amount of outflows driven by sentiment and capable to create a consistent loss (or economic failure) in the market. In periods of liquidity dry-ups or low performance, investors fear funds seizing unit redemptions due to the pressure of fund managers to sell assets in a distressed market at a price smaller than their fair value. In particular Morris and Shin (2004) present a model with short- and long-term investors. They state that irrational behavior and the subsequent creation of a LBH are caused by the fear that asset prices may fall below the limit loss of the short-term investor. When the price decreases significantly, a short-term investor[4] - not knowing the limit loss of other investors - fears the worst loss scenario because other agents may decide to redeem. Therefore investors sell even if their limit is not broken and they cause a liquidity run themselves (even if there would not be rational reasons to explain it). Furthermore Bernardo and Welch (2004) build a model in which the LBH is caused by the fear of investors to have a liquidity shock in a period of a run. In this situation they will not been able to liquidate their assets at a fair value. Therefore, they prefer to sell their assets now instead of facing the possibility of incurring in a sale during the run period. Hence LBHs are not caused by a realized liquidity shock, but by the fear of a possible future liquidity shock. Finally, Huang (2003) presents a model of optimal asset allocation, which takes the possibility of a liquidity shock randomly affecting investors into account. She defines the boundaries for the liquidity premium and the risk-free premium in order to establish a correct asset allocation.

Several studies, instead, identify the uncertainty in the market as the main cause for liquidity imbalances. Uncertainty creates an imbalance in flows and investors' behavior, leading to a reduction in prices driven by illiquidity (i.e. investors would be willing to invest only if there is a reward for their liquidity risk). In accordance with this theory, Easley and O'Hara (2010) affirm that liquidity issues arise because of a non full agents participation in the market. Net flows are then originated from this participation asymmetry and

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[4]If investors were aware of other traders limit loss, than the model will originate multiple equilibria, and will not ultimately lead to a liquidity black hole. This is a fundamental assumptions for the model, and is based on the often limited transparency of the markets.

ultimately illiquidity is brought about by uncertainty due to the lack of trading. Moreover, they observe that illiquidity is caused by uncertainty about future performances and the value of underlying assets. Moreover, Routledge and Zin (2009) show that the uncertainty related to the performance of the underlying financial asset sensibly reduces the liquidity of that asset and consequently of the market. Therefore, crisis are originated for both a liquidity effect (i.e. through the bid-ask spread) and uncertainty. As a consequence, people stop trading and this increases the bid-ask spread, and ultimately leads to a reduction in liquidity.

Other researchers argue that illiquidity and order imbalances are caused by the cost to participate in the market. Potential investors have to face both personal and market constraints. These costs generate imbalances in players' flows and the market moves away from strong and semi-strong efficiency.Therefore, participation costs impede a full participation in the market. Among these researches we can find J. Huang and J.Yan (2007),A. W. Lo and Wang (2001) and Vayanos and Wang (2009). Particularly, Huang and Wang (2009) demonstrate that costly market presence generates a trading imbalance. This imbalance is overwhelmed by the sell side (outflow) and then causes a need for liquidity. This endogenous need can therefore cause a market crash without the necessary condition of a specific aggregate to happened. Moreover, Huang and J.Wang (2010) demonstrate that the cost to participate in the market creates a liquidity issue due to the flow imbalance and, ultimately, it represents a serious problem for the welfare. However they also show that when the cost to participate for a market maker is below a specific threshold, the liquidity problem does not exist because the market making sector is able to absorb order imbalances. Along with this, Brunnermeier and Pedersen (2009) demonstrate that the costs of funding speculators' investments can cause liquidity spirals for two reasons: a funding shock moves the margin and reduces market liquidity, with subsequent further increase in the margin; if a speculator has a significant market share, an induced sale of her assets corresponds to a further price change.

Finally some studies focus on the solution to the illiquidity phenomenon by looking at flows and price movements after and/or during a LBH. This is the case for the phenomena such as flight to quality: in periods when liquidity really matters, the manager and investor tend to move their asset allocation to more liquid assets. This effect could be observable with the increase in price recorded for such instruments. Caballero and Krish-

namurthy (2008) show that during a crisis investors prefer to hold liquid assets, mostly because of a Knightian uncertainty about the future. This uncertainty then causes illiquidity, which originates a flight to quality behavior. In addition Caballero and Krishnamurthy (2008) describe the effect of agents' decisions (for both investors and managers) on the welfare and the optimum central bank reaction, under the assumption of an economy with up to two waves of liquidity shocks. Moreover, Vayanos (2004) develops a model with a fund manager being subjected to outflows generated by random and unexpected personal reasons (e.g. investor liquidity need) and fund performances. As a consequence, the manager prefers liquid assets in periods of market illiquidity because of the higher likelihood of seeing a high request of unit redemptions by investors - i.e. investors are willing to pay a premium to invest in a liquid asset during periods of illiquid markets, as in Acharya and Pedersen (2005). In fact, during periods of high volatility the liquidity premium increases, the beta of asset increases and investors become more risk adverse, hence tending to redeem more.

3 A model of investors' behavior

In this section we model the decision making process of investors in a mutual fund, given their preferences and the state of the economy. We will then characterize the equilibrium and show that a LI can be an equilibrium outcome in certain states of the economy.

3.1 The market

The economy has three dates, $t = 0, 1, 2$. At time 0 the various agents are originated and their portfolios are defined. At time 1 agents choose whether to buy, sell or hold, given the price in the market. At time 2 the economy terminates, final wealth is realized and portfolios are liquidated. In this economy there are only two possible investments choices: a single monopolistic mutual fund and cash which is the numeraire. Cash gives a fixed return equal to 1 at date 2 and hence has no volatility. The mutual fund gives a return equal to N at time 2, where N is normally distributed with mean $\tilde{N}$ and standard deviation σ_N.

3.2 Agents

There are three types of agents in the market: investors that are divided in buyers and sellers and a fund manager that act as a market maker. The economy is populated by a large number I of investors and each investor i, $i = 1...I$, is a price taker. For each agent i, preferences are described by the following constant absolute risk aversion(CARA) expected utility function over her final wealth:

$$u^i = -e^{-\alpha W_2^i}, \tag{1}$$

where W_2^i represents the wealth of agent i at time 2 and α is the coefficient of absolute risk aversion[5].

In order to model trading we distinguish between two types of investors: buyers and sellers, with each type accounting for half of the investors. Buyers and sellers have identical preferences but different initial holdings: buyers are endowed with cash and sellers with shares and the value of their initial endowment is identical.

The return for the investors is realized at time 2 and it is a function of their allocations of cash and shares of mutual fund at time 1. The return N_t of the fund is defined as

$$N_t = y_t + z_t, \tag{2}$$

where y and z represent, respectively, the systematic and the specific component of risk. In particular y is considered the un-diversifiable market risk which defines a specific state of the economy.[6] The value of y varies between negative and positive values and the absence of systematic risk in the market is represented by $y = 0$. The value of y is common knowledge for the investors at time 1. The variable z represents the specific risk of the fund. We assume that each investor does not know the true value of z at time 1 but has an expectation over it. We model investors with heterogenous expectations and this feature will imply the possibility of LBH. Both y and z are normally distributed, with expected value Y and $\tilde{z}$, standard deviation σ_y and σ_z respectively and zero correlation.

More specifically, we interpret the specific risk z to be composed by two components: an exogenous and an endogenous one. The exogenous component includes modifications of the fund's value which are not the direct result of managerial decisions, such as the change in value of a specific asset in the portfolio. These shocks are realized at time 1 and are public knowledge. The endogenous component includes instead the specific actions

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[5]The final wealth of the manager will obviously differ from the investors final wealth.

[6]Changes in inflation or in the general level of rents are examples of factors affecting this variable.

taken by the manager.[7] The manager decides his strategies at time 1 but they become public knowledge only at time 2. Therefore at time 1 an investor does not know the true value of N but forms an individual expectation based on a public signal (exogenous component) and an expectation on the manager's decision (endogenous component).

However, the manager at time 1 decides a price to impose on the fund, and therefore influence market expectations. This price maximizes the manager's utility and it is a function of his expectations over the fund along with the effect on flows of a specific price. In this paper we assume that the manager is aware of the dimension of flows that this price will generate, but this is not always the case in reality. A more naive case will be modeled, following a different maximization criterion, and the two solutions will be compared.

3.3 Participation costs

Following the previous literature[8], we model a participation cost c representing the cost for an investor to buy or sell in the market and we study the implications of its magnitude. Evidence suggests that investor supports her purchase of a financial asset with a specific capital structure. Retail investors appear to invest using an equity-prevailing capital structure while institutional investors appear to leverage their positions in line with the target capital structure of the company. Both equity and debt have a cost that varies according to the wealth of the investor and the status of the economy. Moreover investments in mutual funds are subjected to fees, payed to managers for managing the flows and the fund's structure. Also investing in the secondary market requires the payment of a brokerage fee. The cost c is considered, for simplicity, as a lump sum and it is interpreted as a proxy for all the mentioned costs. It is calculated as a percentage of the value of the initial endowment. Only investors that decide to participate in the market undergo the cost c of participation. We define $\eta_i = 1, 0$ respectively as the choice to whether participate or not in the market, for the investor i. If an investor does not participate, she does not incur any cost. In accordance with Easley and O'Hara (2010), we will show that the cost does not influence the equilibrium price and allocation of the economy but it influences

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[7]Within this component, the capital structure and other specific variables are considered. Marcato and Tira (2010) have shown that some variables, such as leverage, cash and asset concentration, drive the return in REMF and are consequences of specific decisions of the manager.

[8]See for example Huang and Wang (2009, 2010).

the buy-sell-hold individual decisions.[9]

3.4 Timeline

At time 0 buyers and seller are defined and no proper action is taken by the agents. Buyers and sellers differ by whether their initial endowment is expressed in either cash or shares in the fund, but the value of the initial endowment is the same for both classes of agents. We respectively define $\gamma_{i,t}$ and $\theta_{i,t}$ the amount of cash and the quantity of shares, expressed as a percentage of the total outstanding shares, that agent i holds at time t. At time 0 buyers are endowed with a quantity γ_0 of units of cash and sellers with a fraction θ_0 of shares. In order to assure market clearing in the market we impose that each seller holds twice the amount of the per-capita quantity of outstanding shares in the economy. Labeling the per-capita fraction of outstanding shares in the economy by $\bar{\theta}$, we therefore set

$$\theta_0 = 2\bar{\theta}. \tag{3}$$

In order to assure that all the outstanding shares are possibly traded, even without allowing for short sales and leverage, we impose that the unitary price of shares at time 0, P_0, is such that buyers can in principle afford to buy all the shares owned by the sellers, hence we set

$$P_0 = \frac{\gamma_0}{\theta_0}. \tag{4}$$

This implies that the value of the initial endowment of buyers and sellers is identical, i.e. $W_0 = \gamma_0 = P_0\theta_0$. Furthermore we treat cash as the numeraire and hence we impose the equilibrium price $P_0 = 1$. Given that at time 0, before expectations about the future returns are formed, cash and shares have the same expected return, in equilibrium they must have the same unitary price. Figure 1 illustrates the timeline describing the evolution of the economy.

[INSERT FIGURE 1 HERE]

Between time 0 and time 1 the idiosyncratic risk y is revealed and investors observe a public signal v over the specific risk of the fund z. Given this signal, each investor i forms an individual expectation $\tilde{z}_i$ on the specific risk of the fund at time 2. Investors are

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[9]The cost c can be influenced by the manager, i.e. entry/exit fee. In this paper, the cost is exogenously set as the focus of our research is on investors' behavior rather than on the optimal fee policy.

risk averse and decide to trade their position in order to achieve their individual optimal allocation, given their individual expectation of the future value of the fund. At time 1, buyers and sellers trade, given their desired allocation. Naturally, agents are not obliged to invest the full amount of their endowment and an optimal mix of the two assets can be achieved, as well as holding their initial positions. Agents are price-takers and P_1 is the price at which the trades are dealt. We consider both the case of P_1 being the endogenous market clearing price and the case of P_1 being exogenously imposed by the manager. In the case in which we do not allow for leverage or short sales, the buyer (seller) can not increase her endowment of cash (shares) beyond γ_0 (θ_0).

After that trading in the market is completed, every investor holds an optimal allocation of the two assets defined as $A^i(\theta^i, \gamma^i)$. At the final date 2 the value of z is revealed, the final wealth is achieved and all portfolios are liquidated. The final wealth of the investor is determined by the asset allocation A traded at time 1:

$$W_2^i = \theta_1^i P_0(1+N) + \gamma_1^i - \eta_i \bar{\theta} P_0 c. \tag{5}$$

where

$$
\begin{aligned}
\gamma_{B,1}^i &= \gamma_0^i - \theta_1^i P_1, \\
\gamma_{S,1}^i &= (\theta_0 - \theta_1^i) P_1
\end{aligned}
\tag{6}
$$

respectively represent the cash for a buyer and a seller at $t = 1$. Plugging (6) into (5) we obtain the new expressions for the final wealth of a buyer and a seller respectively:

$$
\begin{aligned}
W_{B,2}^i &= \theta_1^i P_0(1+N) + P_0\bar{\theta} - \theta_1^i P_1 - \eta_i \theta P_0 c, \\
W_{S,2}^i &= \theta_1^i P_0(1+N) + (\theta_0 - \theta_1^i) P_1 - \eta_i \bar{\theta} P_0 c.
\end{aligned}
\tag{7}
$$

4   Competitive Equilibrium

By definition, in a competitive equilibrium, P_1 is endogenously determined as the price that equates per capita demand and per capita supply. This condition must be verified for both the assets (cash and risky asset), hence:

$$\frac{1}{I} \sum_{i=1}^{I} \theta_i^* = \bar{\theta}, \tag{8}$$

$$\frac{1}{I} \sum_{i=1}^{I} \gamma_i^* = \bar{\gamma}. \tag{9}$$

At time 1 each investor observes the value of the systematic risk y and a signal v over the specific risk of the asset (exogenous component). We model agents with heterogenous expectations: each investor i has a different expectation over the final value of the mutual fund due to the signal v and due to the expectation of the managerial action. More precisely, we define the expectation of agent i as :

$$E_i[z|v] = \tilde{z}_i. \tag{10}$$

The specific risk z is distributed as a normal with expected value $\tilde{z}$ and standard deviation σ_z. We assume that also the individual expectation $\tilde{z}_i$ is distributed as a normal across agents.

$$\max_{\theta_{1,i}} -e^{-\alpha E[W_2]} \tag{11}$$

Plugging (7) into (1) we find the maximization problem of the agents at time 1:

$$\max_{\theta_{1,i}} -e^{-\alpha * E[\theta_{1,i} P_0(1+y+z)+\gamma_1 - \eta\bar{\theta}P_0 c]}, \tag{12}$$

where γ_1 is different for the buyer and the seller according to (7). Given that both z and y follow a normal distribution and plugging (10) into (12) we obtain that the maximization problem can be re-expressed as:

$$\max_{\theta_{1,i}} -e^{-\alpha * (\theta_{1,i} P_0(1+y+\tilde{z}_i)+\gamma_1 - \frac{1}{2}\alpha\theta_{1,i}^2 P_0^2 \sigma_z^2 - \eta\bar{\theta}P_0 c)}. \tag{13}$$

Using the first order condition we obtain the optimal allocation for each investor i:

$$\theta_{1,i}^* = \frac{P_0 * (1 + y + \tilde{z}_i) - P_1}{P_0^2 \alpha \sigma_z^2}. \tag{14}$$

In a competitive equilibrium, condition (8) must be verified. In order to compute the aggregate demand we compute the average expectation of the specific risk z:

$$\hat{z} = \frac{1}{I} \sum_{i=1}^{I} \tilde{z}_i. \tag{15}$$

Aggregating over the individual demand (14) and using (15) we obtain the expression for the per-capita demand:

$$\frac{1}{I} \sum_{i=1}^{I} \theta_{1,i} = \frac{P_0 * (1 + y + \hat{z}) - P_1}{P_0^2 \alpha \sigma_z^2}. \tag{16}$$

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where $\hat{z}$ represent the average belief of the market concerning the return z. Imposing the market clearing condition (8) we find the competitive equilibrium price:

$$P_1 = P_0(1 + y + \hat{z}) - \bar{\theta}P_0^2\alpha\sigma_z^2. \tag{17}$$

Plugging the equilibrium price (17) in the optimal individual allocation (14) we find the competitive equilibrium individual allocation:

$$\theta_{1,i}^* = \frac{\tilde{z}_i - \hat{z}}{P_0\alpha\sigma_z^2} + \bar{\theta} \tag{18}$$

Few things are worth noticing. First, the participation cost c does not influence the individual desired allocation (14). The reason is that participation costs represent a lump sum fee and therefore influence only the decision of whether to operate on the market but not the individual optimal trade. Second, the first order condition and therefore the individual desired allocation (14) is identical for both a buyer and a seller. The reason is that the optimal mix between shares and cash does not depend on the initial endowment but only on the current equilibrium price of shares P_1 and on the expected return N. Third, P_1 is a relative price given $P_0 = 1$. A current equilibrium price P_1 which is lower (higher) than P_0 expresses the relative expected loss (gain) of the return on the fund's equity relatively to the zero risk-free return on cash.[10] According to (17) the equilibrium price can be expressed as the market expected return on the fund (idiosyncratic and specific risk) at time 2, adjusted for the coefficient of risk aversion α and the per-capita endowment $\bar{\theta}$ of the agents. Increasing number of agents[11] will increase the price in the economy for the demand-supply effect. However, with a continuum of agents (i.e. unlimited agents in the market or $\bar{\theta} \simeq 0$) the price is equal to the market expectations of the return, that is $P_1 = P_0(1 + y + \hat{z})$. Fourth, we notice that the equilibrium allocation (18) is given by the average endowment adjusted for a difference between the belief of investors and the average belief of the market corrected by the risk perceived. In fact an investor with higher expectation than the average belief of the market will own an amount of shares greater than the average holding of the market, and viceversa. Moreover high uncertainty or risk aversion, will decrease the spread over the average endowment.

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[10]The zero risk free return on cash can be naturally interpreted as the return on an inflation adjusted zero-coupon bond.

[11]Given that $\bar{\theta}$ is the average endowment of investors, and knowing that all agents have same preferences, therefore $\bar{\theta}$ represent the number of actors in the market, expressed as a percentage: the smaller is the value of $\bar{\theta}$ and the higher is the number of players inside the economy.

4.1 Optimal participation decision

In this section we study the trading decision of buyers and sellers. Each investor knows the dimension of the idiosyncratic risk y and observes a signal v over the specific risk of the fund z at time 1. Given this information, individual expectations are formed over the final value of the fund at time 2. Given her individual expectation, at time 1, a buyer (seller) decides to either buy (sell) shares in the fund and pay a participation cost or to hold the current cash (shares) portfolio and realize a certain (risky) return. The optimal trading decision can be obtained analyzing the difference of the utility in the two possible scenarios. We define the indirect utility function of holding shares in the fund and holding cash as J_θ and J_γ respectively. Obviously, the utility from participating in the market corresponds to J_θ (J_γ) for a buyer and a seller respectively and the participation cost is sustained only when the actor actually enter in the market(i.e. buyer buying units, seller selling units). It is possible to calculate the difference of payoffs from the two strategies. Given that the investors are risk averse the natural notion for such comparison is the difference in the certain equivalents from the two strategies. We define such difference as the net gain function from participation and we label it $g(.)$.

Proposition 1. *The net gain function from participation for each investor i is:*

$$g_i = -\frac{1}{\alpha} ln \frac{J_\theta^i}{J_\gamma^i}. \tag{19}$$

Proof in appendix 1.

Notice that the gain from participation $g(.)$ is positive if and only if $\frac{J_\theta}{J_\gamma} < 1$. Given the specified utility function, J_θ, J_γ take negative values and therefore $\frac{J_\theta}{J_\gamma} < 1$ when $J_\theta > J_\gamma$. Therefore the gain from participation is positive when the expected utility from holding units of the fund is greater than the expected utility from holding the risk-free asset. Given that J^i decreases in the participation cost c, the higher the costs and the less individuals are willing to enter the market. As J_θ and J_γ are in principle different for buyers and sellers, the impact of the costs may be in principle different on the participation of the two classes of agents.

4.1.1 Optimal decision for the buyer

If a buyer decides not to participate, her final allocation will be given only by her initial amount of cash, hence :

$$\theta_1 = \theta_0 = 0,$$
$$\gamma_1 = \gamma_0 = P_0\bar{\theta}.$$

(20)

If we substitute (20) into the objective function (13) we obtain the indirect utility from non-participating for a buyer:

$$J_{B,\gamma} = -e^{-\alpha\gamma_0}.$$

(21)

If the buyer decides instead to buy some shares in the market, plugging the equilibrium price (17) and the equilibrium allocation (18) in the expected value of (13) we obtain her indirect utility from participating:

$$J_{B,\theta} = -e^{-\alpha(\theta^* P_0(y+\tilde{z}) + 2P_0\bar{\theta} - \theta^*\Delta P - \frac{1}{2}\alpha\sigma_z^*\theta^{*2}P_0^2 - \bar{\theta}P_0 c)}.$$

(22)

4.1.2 Optimal decision for the seller

If a seller decides not to enter the market, her final allocation will be given by the initial amount of shares only, hence :

$$\theta_1 = \theta_0 = 2\bar{\theta},$$
$$\gamma_1 = \gamma_0 = 0.$$

(23)

If we substitute (23) into (13) we obtain the indirect utility from participating for a seller:

$$J_{S,\theta} = -e^{-\alpha(2\bar{\theta}P_0(1+y+\tilde{z}) - 2\alpha\sigma_z^2\bar{\theta}^2 P_0^2)}.$$

(24)

If the seller decides instead to sell some shares in the market, plugging the equilibrium price (17) and the equilibrium allocation (18) in the expected value of (13), we obtain the indirect utility from non-participating for a seller:

$$J_{S,\gamma} = -e^{-\alpha(\theta^* P_0(1+y+\tilde{z}) - \Delta\theta P_1 - \frac{1}{2}\alpha\sigma_z^*\theta^{*2}P_0^2 - \bar{\theta}P_0 c)}.$$

(25)

5 Participation to the market and equilibrium

In this section we analyze the joint behavior of both buyers and sellers. We calculate the number of participants for both classes of agents and we analyze the trading behavior by a numerical simulation.

By definition $z \sim N(\mu_z, \sigma z)$, in our case $\mu_z = \hat{z}$ and therefore $z \sim N(\hat{z}, \sigma z)$. Given the density function of a normal distribution, we compute the number of actors entering in the market. The confidence interval can be obtained from the standard deviation of the distribution and the distance of the event from the average of the distribution. From the properties of a normal we know that the error function is given by

$$erf(\frac{n}{\sqrt{2}}), \tag{26}$$

where n is the number of σ_z used for the confidence interval. Moreover we know that $\bar{\theta}$ represents the average per-capita shares holding in the market and therefore we obtain the number of the participants as:

$$erf(\frac{n}{\sqrt{2}}) * \frac{1}{\bar{\theta}} = \#participants.$$

We substitute the expression of n and obtain that

$$erf(\frac{\beth \pm (\hat{z} - x)/\sigma_z}{\sqrt{2}}) * \frac{1}{\bar{\theta}} = \#participants, \tag{27}$$

where x is given by

$$g(.) = 0|\tilde{z}$$

and $\beth$ is the value which represents the 50% of the fund. The sign of $\beth$ is given by the position of x in relation to $\hat{z}$: in particular, if $x < \hat{z}$, then $\beth$ is positive. When x does not exist, the number of participants in the market is 0%.[12]

We present a numerical example of the buyer's participation in figure 6.

[INSERT FIGURE 6 HERE]

The grey area represents the fraction of buyers for whom $g(.) \geq 0$ and therefore participate to the market by trading cash for shares. In this numerical example, expectations

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[12]Algebraically the participation could also be 100% but we impose a specific positive value of the cost c which excludes such possibility.

are such that the market clearing price at which fund's shares are bought is $P_1 = 1.02$ and the participants in the market are the 39.04% of the buyers' population.

When we apply this equation onto the previous model, we can obtain the participation of the investors and check for liquidity imbalance. In absence of any constraint, the economy is in equilibrium,because the demand of units of the funds equal the supply. This is because the market clearing price is applied in the economy. We do not report the actual figure of our model, because the numbers do not have economic reason. In fact we do not impose any restriction on both leverage and shortage, and the equilibrium is reached by considering extreme positions of investors: investor can leverage even more than 300% of their initial endowment due to their expectations. For this reason, we want to study the case in which restriction are applied , which is more common on the market.

5.1 Constraints

In this economy investors have a fixed wealth m that represents the value of their initial endowment. When restrictions are imposed, neither buyers nor sellers can increase the value m of their initial endowment, however at time 2 an investor may achieve a positive or negative payoff depending on the return on the risky investment. At time 0 the economy is defined as the initial endowment of investors (i.e. cash for buyers and risky units for seller). Therefore we can represent the per-capita endowment of the economy as follows:

$$P_0\theta_0 + \gamma_0 = m, \tag{28}$$

where m is a fixed given quantity. Given the initial endowment, it follows the period 1 budget constraint:

$$P_1\theta_1 + \gamma_0 + \triangle\gamma = m + \triangle P\theta_0. \tag{29}$$

Plugging (28) into 29, we can rewrite the per-capita budget constraint:

$$P_1\theta_1 - P_1\theta_0 + \triangle\gamma = 0 \tag{30}$$

This per-capita condition implies that there is no leverage in the aggregate economy and that the economy is closed.[13] At time 1 investors decide their participation in the market, given the previous budget constraint.

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[13]For a not leverage policy in the economy it is necessary that $P_1\theta_1 - P_1\theta_0 + \triangle\gamma + \bar{\theta}P_0c \geq 0$. Therefore the condition (30) is sufficient to guarantee the condition of no leverage in the aggregate economy.

In the case in which shortage and leverage are not allowed at the individual level, we require some specific conditions. In the case of no shortage, the ownership of units of mutual fund can not be less than 0 at any time. While at time 0 the endowment of risky assets is given by definition, at time 1 for both the actors it must be verified that:

$$\theta_1 \geq 0 \tag{31}$$

The previous inequality guarantees no shortage in the economy. For what concerns the case of no leverage , in addition to (30), the buyer's initial endowment of cash must be greater or equal to the value of the risky stock that she desires to buy at time 1 plus the participation cost:

$$\gamma_0 \geq \theta_1 P_1 + \bar{\theta} P_0 c. \tag{32}$$

In order to avoid leverage on the seller side, it must be verified that $\theta_0 \geq \theta_1$, as in this case the period 1 allocation of shares can not exceed the initial endowment of shares. However an investor could still leverage her position and use the excess cash to increase the return of the portfolio given the sure payoff of 1. The no-leverage condition can not be violated by the Buyer because (30) excludes this eventuality. For the seller instead, the cash at time 1 must not exceed the value of the initial endowment of shares minus the participation cost:

$$\gamma_1 \leq P_1 max(\theta_i \in \Theta) - \bar{\theta} P_0 c,$$

with i=1..I and Θ represent the set of all feasible allocation in the economy. Given that $\theta_0 \geq \theta_1$ implies that $max(\theta_i \in \Theta) = \theta_0$, we have that

$$\gamma_1 \leq P_1 \theta_0 - \bar{\theta} P_0 c. \tag{33}$$

These restrictions are useful to distinguish the agents between buyers and sellers through out the three periods. In fact, shortage may allow buyers to became sellers and leverage may allow sellers to become buyers.[14] We also present the case in which shortage and leverage are allowed.

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[14]Moreover, in the specific case of the Real Estate market short selling is hardly practised because of the limited secondary market. Gearing, instead, is possible in the RE unlisted market but it is not very popular among investors (while it is for the funds): REMF investors are mainly institutional investors and also among retailers investor, REMF is still considered a low grade risk investment. Gullet and Redman (2005) demonstrate that RE it is used to reduce the volatility of mixed portfolios and is considered as an inflation-hedge investment.

5.2 Equilibrium with constraints

In this section we want to show the effect of leverage and shortage constraints on the optimal equilibrium of the economy. We have already observed that if we apply the optimal price in the economy, the demand of units from investors equal the supply. This effect is guaranteed by the contemporaneous possibility to leverage and to shortage for both class of investors, without any boundary on their positions. This case is not close to the real scenario, as it is difficult to think that some investors are allowed to leverage more than 3 or four times their initial position. Moreover the specific case of the Real Estate market does not allow to shortage on units in a mutual fund[15]. In addiction INREV shows that the vast majority of investor that approach REMF are institutional and their portfolio is mainly equity, i.e. they do not leverage their position. However in our simulation we allow for a small percentage of leverage as some funds are open to retail investors, which approach REMF with various capital structure. In the following table we illustrate the effect of constraints over the optimal equilibrium:

[INSERT FIGURE HERE]

We did not include the table showing the participation in the market, as the optimal case scenario present an equal participation for both the agents, at any specific return of the fund N, and a net participation equal to 0, i.e. demand equal supply.

From the table 2 we can observe that the introduction of constraints creates imbalances of flows. In particular we observe that increasing returns provide an increasing negative outflow. This counterintuitive effect is driven by the increasing optimal price. In fact the endowment of cash of buyers does not change during time and therefore their ability to buy units of the fund diminishes with increasing prices. Moreover Netflows turn into positive for optimal prices of $P_1 \leq P_0 - c$.

However if we include the possibility of leverage we observe that Netflows are increasing, driven by the increasing purchase power of buyers. In particular, increasing level of leverage increase the Net flows.

We can conclude that if leverage is considered in an optimal price scenario, net flows are directly related to the level of gearing allowed. The final figure of flows is conditional to

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[15]It is possible to shortage units of fund, through derivatives based on the IPD index. However, it is not possible to physically short on the fund.

the optimal price and the cost to participate in the market[16].

5.3 Equilibrium with imposed price

In this section we analyze the model using a price imposed by an external source rather than an optimal combination of sellers' and buyers' preferences. In the mutual fund industry, a transaction price-based approach is the most common: funds are listed in the stock market and available to retail investors. However the case of unlisted mutual fund is different. In this market, the fund manager decides herself a price for each unit (normally by following the theoretical definition: $\frac{NAV}{\#ofunits}$ plus a spread). Hence, in this market context (well represented by REMFs), a model with an imposed price rather than an equilibrium price is more appropriate. In order to isolate the manager's solution to imbalances in order flows, in this paper we consider the mid price of a fund rather than the bid-ask spread.

Obviously the exogenous imposition of the price can cause sub-optimal behaviour of the agents in the economy, leading to imbalances in flows. However the optimal price could be difficult to be achieved in a real case scenario. Therefore the imposed price solution is in general a more realistic case when applied to a real sample.

The price for the unlisted market is generally higher than the transaction-based price. Firstly, there are less units in a fund, by comparing funds of comparable size. Secondly, unlisted funds are less subjected to frequent withdrawals, mainly for two reasons: frequent modification of the structure of the fund are not cost efficient, and the manager tries to avoid them with appropriate pricing policy. Thirdly, investors in unlisted funds are mainly institutional investors with a long-term investment horizon, and provide their liquidity through their asset allocation.

We repeat the numerical analysis for the case of fixed price using a given price of $P_1 = 0.9$[17]. We substitute such value to (17) in (19) and we obtain the expression of the net

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[16]It must be noticed that the cost to participate in the market include the price of funding. Therefore it is likely to assume that the cost of participation is increasing in the level of gearing. In our research we do not take this issue into account in order to simplify the model.

[17]The price is simply chosen as the average of the optimal prices considered in the previous example.

gain function for both buyer and seller:

$$g_B(y, \tilde{z}) = -\frac{1}{\alpha} ln\left(\frac{exp[-\alpha(3\theta^*(y + \tilde{z}) + 2P_0\bar{\theta} - 4.5\alpha\sigma^2\theta^{*2} - c)]}{exp[-2\alpha P_0\bar{\theta}]}\right),$$

$$g_S(y, \tilde{z}, \hat{z}) = -\frac{1}{\alpha} ln\left(\frac{exp[-\alpha(6\bar{\theta}(y + \tilde{z}) + 2P_0\bar{\theta} - 18\alpha\sigma^2\bar{\theta})]}{exp[-\alpha(3\theta^*(y + \tilde{z}) + 6\bar{\theta} - 4.5\alpha\sigma^2\theta^{*2} - c)]}\right).$$

Given the two net gain functions, we compute the market participation and the flows for different values of returns. We present the results in the figure:

[INSERT FIGURE HERE]

In aggregate, we notice that the volume of the outflows dominates the volume of inflows, therefore there is an imbalance. Particularly the net flows turn into negative if:

$$P_{optimal1,N} < P_{imposed1} \tag{34}$$

This imbalance is due to the certainty equivalence of agents for specific states of the nature. Moreover it is worsen by asymmetric impact of the participation cost c and of the leverage-shortage constraints on buyers and sellers.

5.4 Comparative Statics

In order to gain a better economic intuition of the behavior of the gain function, we compute its partial derivatives with respect to the function variables.

For the buyer case, we notice from the expression of (22) that the net gain is affected by the spread between y and $\tilde{z}$, but with opposite signs, hence the overall effect is not a priori clear.

[INSERT FIGURE HERE]

Figure 4 shows indifference gain curves in terms of y and $\tilde{z}$. Three different areas are easily recognizable. In the shortage area, i.e. with negative values of z, the net gain of participation is negative and is recognizable as a flat surface. When the $g(.)$ function hits the investment area, i.e. between the two imposed boundaries, it grows with a concave slope. Above the leverage area, the net gain function grows with a constant slope: the endowment of units in the investor's portfolio can not increase over the leverage limit, but augmenting values of z boost the final gain for the investor. Increasing values of the

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net gain are given by increasing values of both y and $\tilde{z}$ in the other two areas. This verifies that the single components of risk y and $\tilde{z}$ do not affect the net gain function per se, but it is rather the overall expected return of the fund to influence the decision to wether buy or not. We also notice from (22) that, given the coefficient risk aversion α, increasing volatility σ_z decreases the net gain of participation.

Also for the case of the seller, we notice from the expression of (24) that the net gain is affected by the spread between y and $\tilde{z}$, but with opposite signs, hence the overall effect is not a priori clear. In this case it is the indirect utility of participating (24) to be affected because it is by participating that the seller holds more shares and therefore is more affected by the specific risk. As for the buyer case, we plot indifference gain curves in terms of y and $\tilde{z}$.

[INSERT FIGURE HERE]

Also in the seller case the single components of risk y and $\tilde{z}$ do not affect the net gain function per se, but it is rather the overall expected return of the fund to influence the decision to wether sell or not. The net gain function still presents three distinct areas. The leverage area is represented as a flat surface, recalling the case of the buyer 's shortage area. However, the investment area decreases with a higher slope than for the buyer's case. This result suggests that selling is preferable to holding even for low positive returns of the fund. In fact the risk averse investor prefers to hold a positive amount of risk-free asset, i.e. cash, rather than to be exposed to the uncertainty of the risky one, for low positive return and average prices. Finally in the shortage area, the net gain function decreases at a constant rate, but this is smaller than the rate at which the buyer's net gain function increases in the leverage area. This asymmetric behavior between buyers and sellers suggests the possibility of an imbalance in the flows of the fund.

5.5 The impact of costs and uncertainty over flows

In this section we provide a further analysis on two main drivers of liquidity imbalances: costs and uncertainty. We want to study the impact of a variation of these two variables on the flows in the fund. These variables has been dictated as critical by the literature, and our model provides a serious tool to quantify their impact.

The cost to participation, as previously illustrated, is the main source of liquidity imbalance according to Huang and J.Wang (2010). In fact it moves the investors away from the fully participation in the market, it reduces the returns for both the agents and it has a

direct effect on the purchase power of buyers. In this research, the participation cost is expressed as a lump sum on the initial endowment of an agent,i.e. as a percentage. It depends only on his decision to participate and not on his final optimal allocation. In figure 5.4 we give an example of how the net gain function changes as a function of $\tilde{z}$, given different values of the cost c.

[INSERT FIGURE HERE]

As we can observe from the figure, the participation costs is influencing both the participation decision and the flow dimension, as expected. Panel A and B show that an increase in costs move the participation of agents. They need to achieve higher expected return in order to obtain a non negative utility function, and therefore a convenience in sharing their risk exposure on the market. Moreover, the cost does not effect the optimal allocation of investors as expressed by (17) and (18), but influence the purchasing power of buyers and therefore, if leverage constraint is applied, it reduces the inflows in the fund[18]. This originate a liquidity imbalance in the fund. The dimension of the imbalance grows for increasing participation costs, as can be seen from the panel C. We also verify, as proven by Huang and J.Wang (2010), that for a null participation cost $c = 0$ everyone is always entering the market.

The uncertainty in this model is considered under the variable $\sigma_{\tilde{z}}^2$. It accounts for the range of decisions that every agent in the economy should take. An increase of uncertainty in the system increase the single choice volatility over the investor optimal decision. This effect is driven by the uncertainty over the expected return, through the expectation he is creating on the signal S, and is therefore an uncertainty over the information inside the economy.
Many studies has been developed on the impact of uncertainty over decisions. In our research we want to study the effect of a variation of uncertainty over the single investor decision and the impact on his allocation.
We run the model using different σ in the economy and we account for changes in flows. We use the imposed price model without restriction mainly for two reasons: first of all it is the only model that allow for imbalances in flows,that are the main target of this test. Secondly this case is closer (than the imposed price with restrictions) to a general market

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[18]In absence of Leverage constraint the cost c has no effect on inflows. For further details on cases in which this constraint is applied please refer to the previous section.

case, and it can provide more realistic figures. The results of this analysis are illustrated in the following three graphs:

[INSERT FIGURE HERE]

Change in Net-,in-, and outflows has been reported in the table. As we were expecting, increasing value of σ_z^2 lead to a decrease of flows. When market information are scarce, expectancies become less reliable, and investors prefer not expose theirselves to the risky asset due to their risk averse nature. Moreover all three different flows have a linear growth. We would expected an exponential growth given the CARA preference used to describe the behaviour of the agents, but the restriction on leverage and shortage reduce this effect. Moreover, flows rotate over a specific negative point (i.e. when $P_i mposed = P_o ptimal \pm c$) and increase in accordance to their certainty equivalence. If the expected return is overpriced on the market, investors do not have any interest in buying unit of the fund, and the outflow is prevailing: inflows turn to 0, and net flows are linearly decreasing.

The risk aversion nature of investors combined with the uncertainty in the market could lead to an un-welcomed flow imbalance even for specific positive states of the nature. For this reason we want to analyze the effect on flows of a change in σ_z^2 keeping constant the signal in the economy and consequent mismatching in behaviors that will originate.

In our model we account for a level of uncertainty that is equal for all the agents. However investors can have different uncertainty over their expectations[19] and different scenarios can be originated. The asymmetry of information is proven to exist into financial market, but it is trivial to measure the level of uncertainty for the single agent. Therefore our model will use a collective measure of this value, as approximation of the behaviour of the single agent.

In this graph we analyze the flows variations due to investors with different uncertainty. This help to draw the liquidity imbalances driven by information asymmetry that may originate in the market. In order to implement this test we run the model keeping constant the expected return, and modifying the uncertainty of investors. The following graph is illustrating the results:

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[19]The non-efficiency of the market driven by asymmetry of information has already been deeply analyzed and proven by numerous papers in the finance and economics literature. For this reason we will not analyze further in this research but we take this information as granted.

[INSERT FIGURE HERE]

From the table we can observe that flows are growing for decreasing value of σ_z^2 with an exponential growth. The shape of the growth is connected with the preference form of agents in the economy. However the concavity is decreasing for decreasing expected return, changing for return next to the turning point, as previously observed. As a whole, flows are increasing with the decreasing of the uncertainty in the market. Certainty over future expectations drive investors to move in the same direction, and therefore to increase their allocations.

Finally, for a high level of uncertainty in the market the flows are negative even for positive state of the nature, i.e. $\hat{N} = 10\%$. This counterintuitive effect is driven by the certainty equivalence. Increasing uncertainty in the market influence the optimal price of the economy due to a modification in the demand-supply mechanism. Therefore a negative net flow could be achieved even if the expectation in the market are positive. These imbalances can then lead to a LBH. As a consequence the manager can not hedge against negative flows with an appropriate pricing policy unless he decide to undergo a significant reduction on returns. Therefore a LI becomes an un-hedgeable risk even in positive states of the nature. The current state of the economy is experiencing something similar: financial assets with positive predictions are experiencing negative outflows.

6 Real Case scenario

As an empirical application of our theoretical model, we apply it to a real case scenario, using a specific REMF established in the United Kingdom. We have chosen to apply the model to the Aviva investor pension fund, taking data from the IPD property pooled fund database. The data sample is collected quarterly between March 1995 and March 2009. The large sample allows us to set up and tune the model, although the latter is applied only on the period 2005-2009. We decided to study this fund for three main reasons: availability of data, high number of participants (around 150) and limited percentage of fund owned by internal investors. The expectations of investors are distributed as a normal distribution and the high number of agents allow to get closer to this distribution. Moreover the internal investors are not modeled in our case and the results could be biased by this.

Every quarter is to be considered as an economy, and therefore investors will liquidate

their position in the following instant. The idea is that the characteristics of the previous period define the values of variables in the following period. REMFs' investors are mainly institutional investors and are not looking for an active trading in the fund, but rather they will withdraw their positions when the future expectancies are not favorable.

In the simulation we allow for different scenarios of uncertainty in the market. In fact we think that the RE market has experienced different expectations during the sample considered with an increase of uncertainty towards the second half of 2007 and immediately after the Lehmann brothers' collapse[20]. Moreover we expect two different average level of leverage in the market. The first part of the sample ($2005 - 2007$) represents part of the boom cycle of the RE market, while in the second part bank has tighten their credit support, and therefore leveraging position for investors has become increasingly difficult. For this reason we consider a level of leverage of 10% in the first part of the sample and no leverage in the second part of it.

The variable $\hat{z}$ is the total return of the previous period. The population of the fund is the number of investors at the end of the previous quarter, after in and outflows, but it considers the agents that are waiting to redeem their units, i.e. redemptions outstanding. The costs for both buyers and sellers include a cost for brokerage and for fees and taxes, on top of the value declared by the fund. The cost is expressed as percentage of the initial endowment.

The manager apply a bid and ask price to the fund at each quarter. Therefore we will apply the model in the imposed price scenario using the mid price of the previous quarter from the report of the fund. This paper is not aiming at imposing an optimal pricing policy on the market, but simply at forecast the dimension of flows in a fund.

The following figure is representing the model applied on the available data and the actual figure of net flows in the fund:

[INSERT FIGURE HERE]

From the picture we can observe that the model is able to predict the direction of the flows in the fund. However the differences between forecasted values and the actual figures are given by the approximation of the model both theoretical and empirical. Moreover we think that the REMF market is influenced by irrational behaviour of investors. Despite the vast majority of REMF investors is institutional,i.e. tend to trade only under specific rational behaviour , the assumption of complete rational market can not be

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[20]We run the model also with a uncertainty obtained from historical volatility data, but the results does not change significantly.

proven.

In the first part of the sample the value of predicted flows is very close to the actual figure. The forecast power of our model is helped by the returns that are held constants on positive levels,i.e. around 4%, and the low uncertainty levels on a rising market. After Q3 2007 we can observe that predicted and actual flows differ significantly even if they keep the same slope,i.e. they rise and fall for the same quarters. The highly negative pick in Q2 2008 is driven by reasons not considered in the model, such as irrational behaviour or a significant change of the internal structure of the fund[21]. Notwithstanding the high uncertainty in these periods, these results suggest that the overall effect is driven by more than this variable alone.

7   Conclusions

We presented a model of optimal behavior for mutual fund investors, and we explicitly modeled the origination of a liquidity imbalances. The investor's choice (i.e. buy-sell-hold) is based on CARA preferences and is a function of the idiosyncratic risk and the expected return, given a signal on the specific risk. The result of the model is based on expectations of both market and investors. The analysis of investors' behavior is crucial to understand irrational consequences connected to liquidity dry-ups in the economy. Particularly, an attentive study on the reasons behind movements in the decision function allows the manager and the market maker to limit the change in fund pricing.

We define the conditions for the creation of a liquidity imbalances focusing on the participation of investors in the market and the capacity of the market maker to hedge the problem of redemptions.

Starting from the liquidity effect on asset pricing and following an intuition in Marcato and Tira (2010), we discover that illiquidity is originated at a micro-structural level rather than in a macro-economic context. Specifically, looking at our current state of the economy, we discover that the optimal price and allocation are the same for both buyer and seller, in line with Easley and O'Hara (2010). Furthermore, we discover that the cost to participate in the market does not influence the optimal allocation but has a significant effect on the investor's decision, preventing the full participation, as illustrated by Huang and J.Wang (2010). The investor's decision is influenced by the unit price, especially for

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[21]As previously observed our model is aim to describe only the rational behaviour of agents in the market.

the seller. We create a net gain function which models investors' choices. We then apply it to a randomly generated scenario. The result is a map of investor's behaviors, given different levels of risk and expectations.

In addition, participation rules are applied to the function, and the dimension of REMF flows is derived. A close approximation of the number of participants in the market (expressed both as a percentage and number of agents) is obtained. More specifically, if an optimal price is imposed, investors' participation is equal for both buyers and seller. However the outflows effect prevails, driven by a difference in purchasing power of the two agents in the economy.

Furthermore, we created a more general case, in which we impose the price of the fund to be an exogenous information. Comparing our results with the previous model, the participation of buyers increases when there is a higher price per unit, and decrease otherwise. On the other hand, the seller is subjected to a fixed price and therefore she experiences a different behavior driven by similar preferences. We prove that imbalances in flows can be originated in this scenario and are worsen in areas with significant negative values for both components of risk.

Moreover we discover that an increase of uncertainty in the market diminish the magnitude of flows and increase the certainty equivalence. Ultimately it can lead to liquidity imbalance even for positive state of the nature.

Finally we applied our model to a real case scenario and the results are consistent with the data observed, corrected for irrational behaviours in the market.

In conclusion, LIs do exist and could be originated either by a particular negative state of the nature along with a wrong pricing policy from the manager , by the cost to participate in the market, by imposing of restriction on the capital or from an elevated degree of uncertainty in the economy. Depending from the source of the origin, the imbalance could be hedged or a more drastic solution is required.
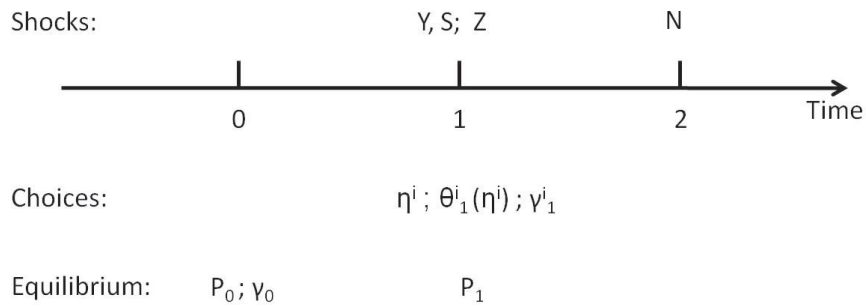
Figure 1: *Timeline of the economy. Shocks are endogenous to actors, Choices are decided by actors, and Equilibrium is a consequence of actors' choices*
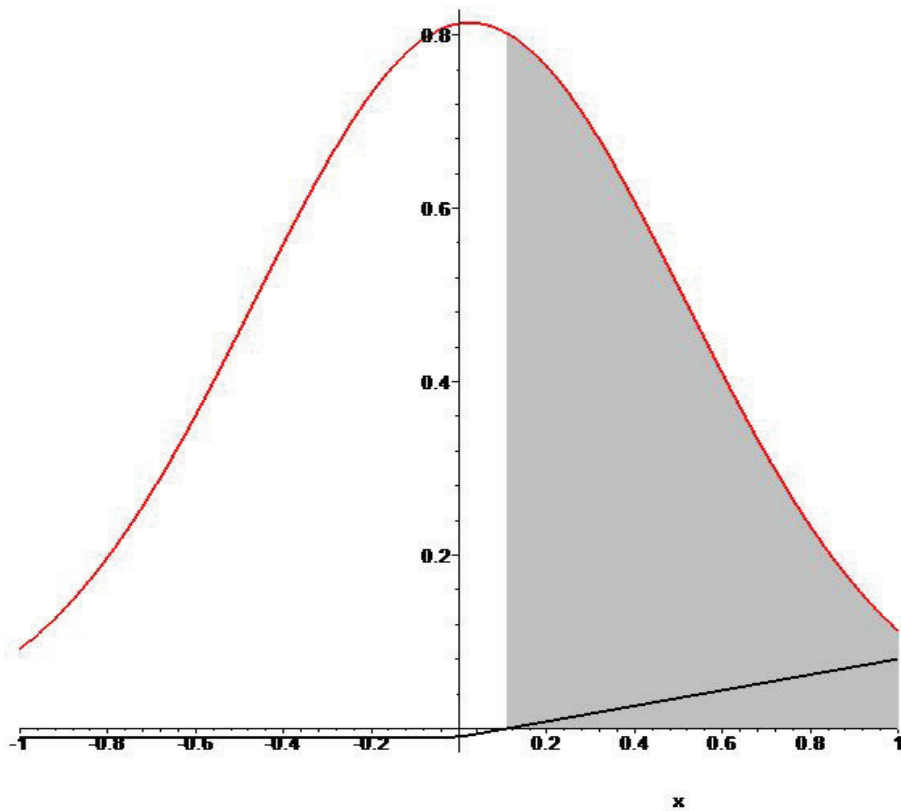

Figure 2: *Buyer Participation function. The grey area represent the area of participation for buyers.The x axis represents the expectations z of investors. Variable used for the graph:* $\alpha = 4, \sigma = 0.7, \bar{\theta} = 0.05, \hat{z} = 0.05, y = 0.03, c = 0.01, P_0 = 1$

N	P	BUYER	SELLER	Inflows	Outflows	Net flow	LEV 10%	LEV 30%
15.0%	1.14	30.20%	31.30%	25.00%	31.30%	-6.30%	*-3.3%*	*7.5%*
10.0%	1.09	30.20%	31.30%	26.15%	31.30%	-5.15%	*-2.0%*	*9.3%*
5.0%	1.04	30.20%	31.30%	27.41%	31.30%	-3.90%	*-0.6%*	*11.2%*
1.0%	1.00	30.20%	31.30%	28.50%	31.30%	-2.80%	*0.6%*	*12.9%*
-1.0%	0.98	30.20%	31.30%	29.08%	31.30%	-2.22%	*1.3%*	*13.8%*
-5.0%	0.94	30.20%	31.30%	30.32%	31.30%	-0.98%	*2.7%*	*15.8%*
-10.0%	0.89	30.20%	31.30%	32.02%	31.30%	0.72%	*4.6%*	*18.4%*
-15.0%	0.84	30.20%	31.30%	33.93%	31.30%	2.63%	*6.7%*	*21.4%*
-30.0%	0.69	30.20%	31.30%	41.31%	31.30%	10.01%	*15.0%*	*32.8%*

Table 1: *Buyers and sellers flows, optimal price case scenario with shortage and leverage constraints. Buyers and sellers column represents the participation in the market. Case with 10% and 30% leverage allowed. Variable used for the table: $\alpha = 4, \sigma = 0.7, c = 0.09, \theta = 0.005, P_0 = 1$*

N	BUYER	SELLER	Inflows	Outflows	Net flow
15.0%	36.5%	26.2%	31.6%	26.2%	5.4%
10.0%	29.0%	29.0%	29.1%	29.0%	0.1%
5.0%	30.8%	31.9%	26.7%	31.9%	-5.2%
1.0%	28.5%	34.1%	24.7%	34.1%	-9.4%
-1.0%	27.4%	35.3%	23.7%	35.3%	-11.6%
-5.0%	25.1%	37.5%	21.8%	37.5%	-15.7%
-10.0%	22.4%	40.4%	19.4%	40.4%	-21.0%
-15.0%	19.7%	43.2%	17.1%	43.2%	-26.1%

Table 2: *Buyers and sellers flows, imposed price case scenario with shortage and leverage constraints. Buyers and sellers column represents the participation in the market. Variable used for the table: $\alpha = 4, \sigma = 0.7, c = 0.09, \theta = 0.005, P_0 = 1, P_1 = 1.05$*
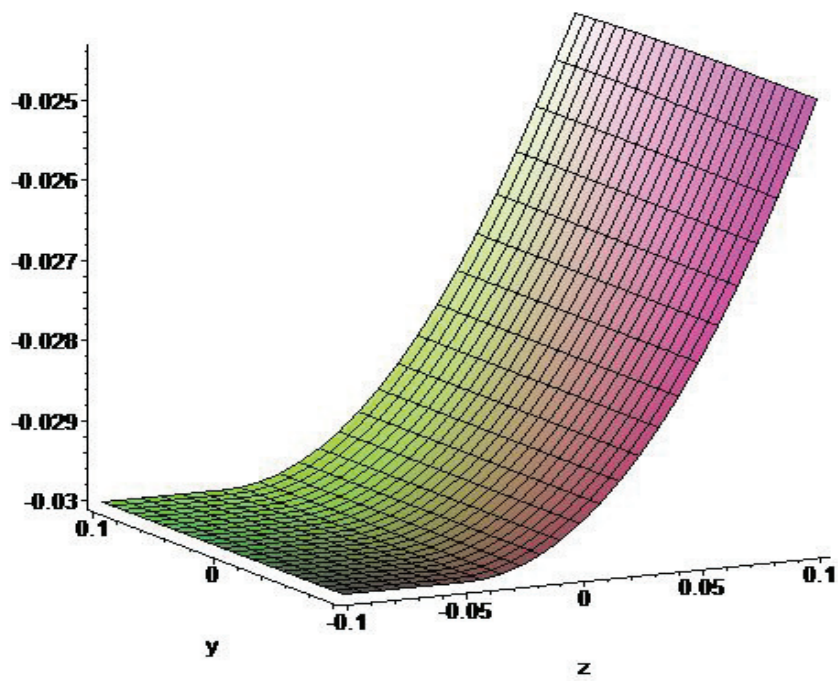
Figure 3: *Buyer net gain function given value of y and z with shortage and leverage constraints. Indifference curves grow from green to purple. Variables used for the graph:* $\alpha = 4, \sigma = 0.7, c = 0.03, \theta = 0.05, P_1 = 1$
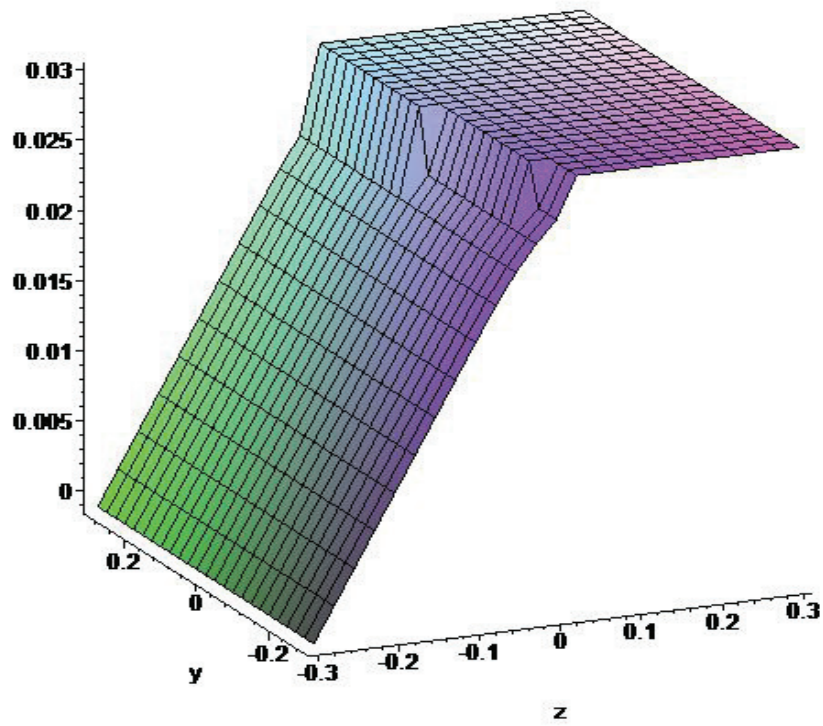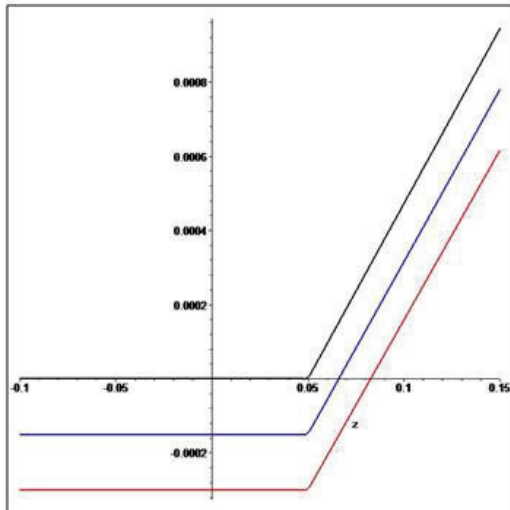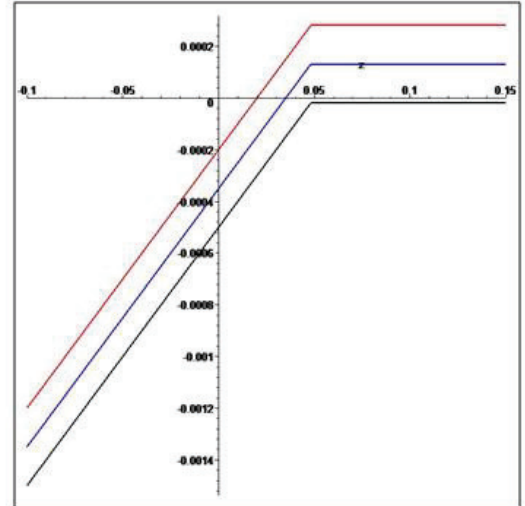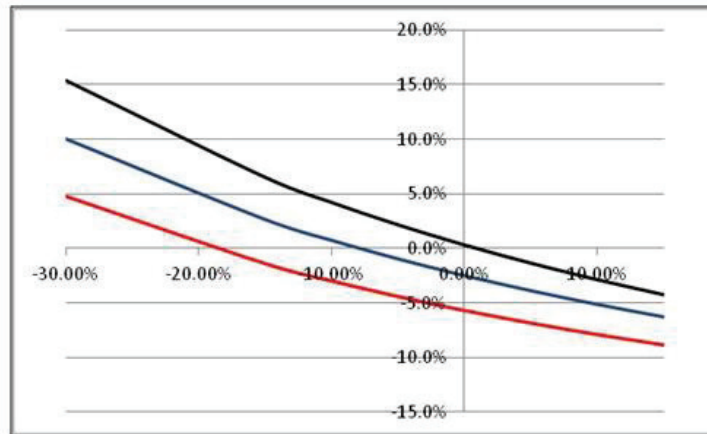
Figure 4: *Seller net gain function given value of y and z with shortage and leverage constraints. Indifference curves grow from green to purple. Variable used for the graph:* $\alpha = 4, \sigma = 0.7, \hat{z} = 0.05, c = 0.03, \theta = 0.05, P_1 = 1$

**Panel A: Buyers**

Panel B: Sellers

Panel C: Netflows

Figure 5: *Panel A:Buyer net gain function with different costs to participate in the market, shortage and leverage constraints. Panel B: Seller net gain function with different costs to participate in the market, shortage and leverage constraints. Panel c: Net flows with different costs to participate in the market, shortage and leverage constraints.Variable used for the graph:* $\alpha = 4, \sigma = 0.3, \theta = 0.005, y = 0.01, \hat{z} = 0.05, P_0 = 1, c_{black} = 0, c_{blue} = 0.09, c_{red} = 0.2$
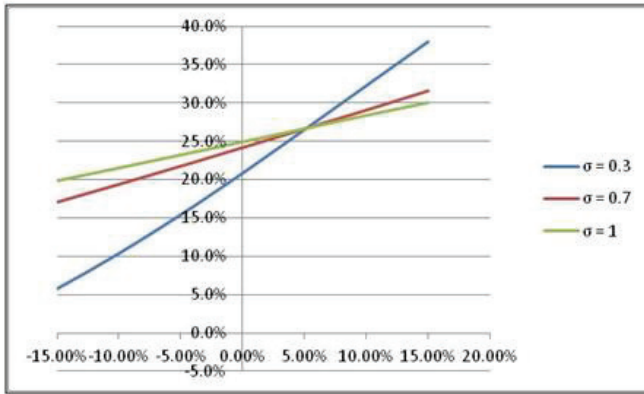
Panel A: Inflows
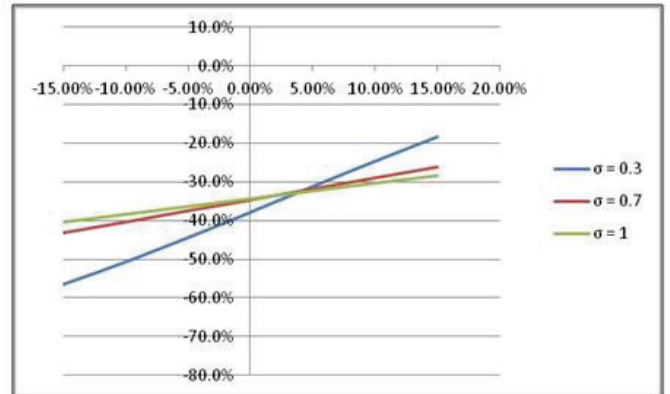
Panel B: Outflows
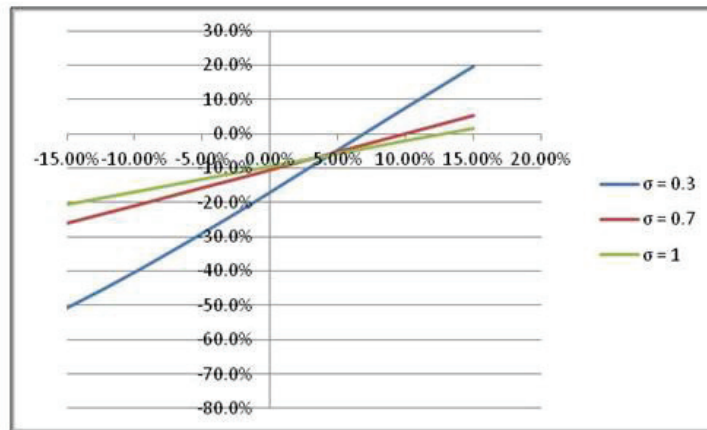
Panel C: Netflows

Figure 6: *Inflows, Outflows, and Net flows given different uncertainty level in the market. Variable used:*
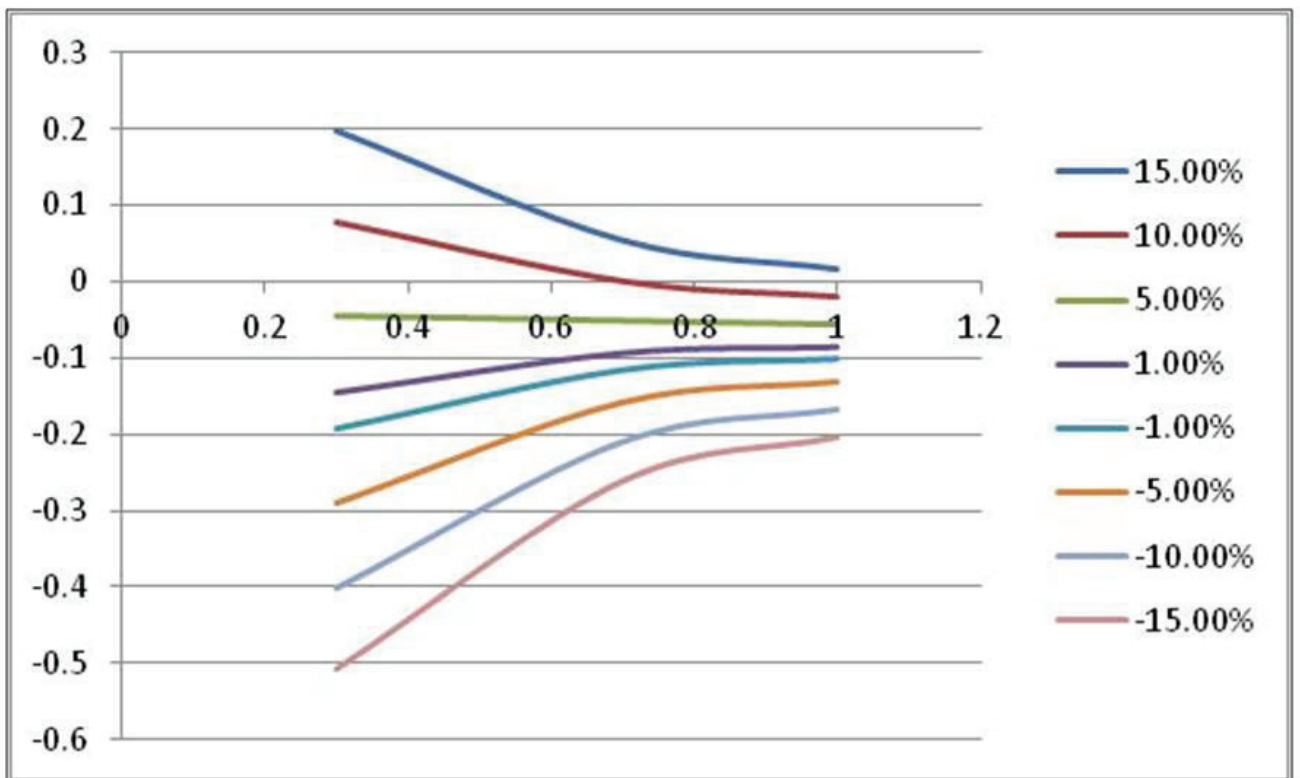$\alpha = 4, \theta = 0.05, P_0 = 1, c = 0.09$

Figure 7: *Net flows given different uncertainty level in the market. Every line represent a specific expected return $\hat{z}$. Variable used:* $\alpha = 4, \theta = 0.05, P_0 = 1, c = 0.09$
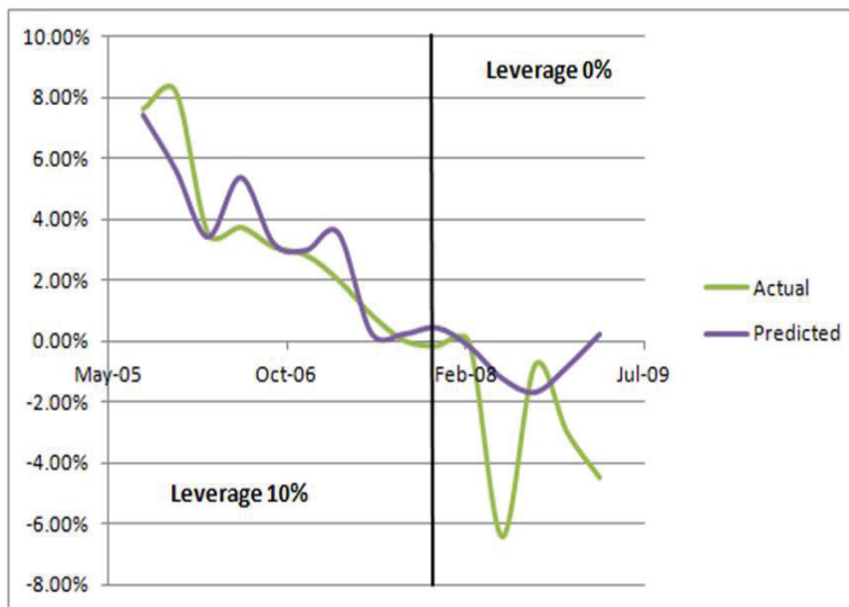
Figure 8: *Aviva investors pension: real figure vs. forecast model. The green line represents the actual flows in the fund while the purple line are the predicted flows. Variable used for the graph:* $\alpha = 4, \sigma = 0.7, P_0 = 1, c = 0.09$

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Appendix 1

Proof of proposition 1

The certainty equivalent is defined as the wealth W^* such that an investor is indifferent between participating in a gamble and receiving W^* with certainty. Given the chosen CARA utility function the certainty equivalent from participation can be found solving the following equation:

$$e^{-\alpha W_p^*} = J_p.$$

Therefore we find that $W_p^* = -\frac{1}{\alpha} \ln(J_p)$. Symmetrically we find that the certainty equivalent from non participation is $W_{np}^* = -\frac{1}{\alpha} \ln(J_{np})$. Subtracting we find that $W_p^* - W_{np}^* = -\frac{1}{\alpha} \ln\left(\frac{J_p}{J_{np}}\right)$.